

Reg. No. \_\_\_\_\_ Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
THIRD SEMESTER B.TECH DEGREE EXAMINATION, DEC 2016

**Course Code: MA201**

**Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS**

Max. Marks: 100

Duration:3. Hours

**PART A**

**(Answer any two questions)**

- 1.a Show that  $u = y^3 - 3x^2y$  is harmonic and hence find its harmonic conjugate. (8)
- b Find the image of  $\left|z - \frac{1}{2}\right| \leq \frac{1}{2}$  under the transformation  $w = \frac{1}{z}$ . Also find the fixed points of the transformation  $w = \frac{1}{z}$  (7)
- 2.a Define an analytic function and prove that an analytic function of constant modulus is constant. (8)
- b Find the linear fractional transformation that maps  $z_1 = 0, z_2 = 1, z_3 = \infty$  onto  $w_1 = -1, w_2 = -i, w_3 = 1$  respectively. (7)
- 3.a Show that  $f(z) = e^{-x}\cos y - ie^{-x}\sin y$  is differentiable everywhere. Find its derivative. (8)
- b Find the image of the lines  $x = c$  and  $y = k$ , where  $c$  &  $k$  are constants, under the transformation  $w = \sin z$ . (7)

**PART B**

**(Answer any two questions)**

- 4.a Evaluate  $\int_C \operatorname{Re}(z) dz$  where  $C$  is a straight line from 0 to  $1 + 2i$ . (7)
- b Show that  $\int_0^\infty \frac{dx}{1+x^4} = \frac{\pi}{2\sqrt{2}}$  (8)
- 5.a Integrate  $\frac{z^2}{z^2-1}$  counterclockwise around the circle  $|z - 1 - i| = \frac{\pi}{2}$  by Cauchy's Integral Formula. (7)
- b Evaluate  $\int_C \frac{z-23}{z^2-4z-5} dz$  where  $C$  is  $|z - 2 - i| = 3.5$  by Cauchy's Residue Theorem (8)
- 6.a If  $f(z) = \frac{1}{z^2}$  find the Taylor series that converges in  $|z - i| < R$  and the Laurent's series that converges in  $|z - i| > R$ . (8)
- b Define three types of isolated singularities with an example for each. (7)

## PART C

(Answer any two questions)

7.a Solve by Gauss Elimination:

$$\begin{aligned}x_1 - x_2 + x_3 &= 0, \\-x_1 + x_2 - x_3 &= 0, \\10 x_2 + 25 x_3 &= 90, \\20 x_1 + 10 x_2 &= 80.\end{aligned}\tag{5}$$

b Find the rank. Also find a basis for the row space and column space for

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -4 \\ 0 & 4 & 0 \end{bmatrix}\tag{5}$$

c Find out what type of conic section the quadratic form

$$Q = 17 x^2 - 30 xy + 17 y^2 = 128 \text{ represents and transform it to the principal axes.}\tag{10}$$

8.a Find whether the vectors  $[1 \ 2 \ -1 \ 3]$ ,  $[2 \ -13 \ 2]$  and  $[-1 \ 8 \ -9 \ 5]$  are linearly dependent. (5)b Show that the matrix  $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$  is symmetric. Find the spectrum. (5)c Diagonalise  $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$  (10)9. a. Determine whether the matrix  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$  is orthogonal? (5)b. Find the Eigen values and Eigen vectors of  $\begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$  (5)

c. Define a Vector Space with an example. (10)