Reg. No.____________ Name:________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FOURTH SEMESTER B.TECH DEGREE EXAMINATION, MAY 2017

MA202: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A (MODULES I AND II)

Answer two full questions.

1. a. Given that $f(x)=\frac{k}{2^x}$ is a probability distribution of a random variable that can take on the values $x=0,1,2,3$ and 4, find k. Find the cumulative distribution function. (7)

   b. If 6 of the 18 new buildings in a city violate the building code, what is the probability that a building inspector who randomly select 4 of the new buildings will catch

   i) none of the new buildings that violate the building code

   ii) one of the new buildings that violate the building code

   iii) at least two of the new buildings violate the building code (8)

2. a. Prove that binomial distribution with parameters n and p can be approximated to Poisson distribution when n is large and p is small with $np=\lambda$ a constant. (7)

   b. Find the value of k for the probability density $f(x)$ given below and hence find its mean and variance where

   $$f(x)=\begin{cases} kx^3 & 0<x<1 \\ 0 & otherwise \end{cases}$$

   (8)

3. a. A random variable has normal distribution with $\mu=62.4$. Find it's standard deviation if the probability is 0.2 that it will take on a value greater than 79.2 (7)

   b. The amount of time that a surveillance camera will run without having to be reset is a random variable having the exponential distribution with the parameter 50 days. Find the probability that such a camera will

   i) have to be reset in less than 20 days

   ii) not have to be reset in at least 60 days. (8)

PART B (MODULES III AND IV)

Answer two full questions.

4. a. Use Fourier integral to show that $\int_0^\infty \frac{\cos x\omega+\omega\sin x\omega}{1+\omega^2}d\omega=\begin{cases} 0 & if\ x<0 \\ \pi/2 & if\ x=0 \\ \pi e^{-x} & if\ x>0 \end{cases}$ (7)

   b. Represent $f(x)=\begin{cases} x^2 & 0<x<1 \\ 0 & x>1 \end{cases}$ as a Fourier cosine integral. (8)

5. a. Find the Fourier transform of $f(x)=\begin{cases}1 & if\ |x|<1\\ 0 & otherwise\end{cases}$ (7)

b. Find the Laplace transforms of the following

i) $\cos t - t\sin t$ ii) $4t\,e^{-2t}$ (8)

6. a. Find the inverse Laplace transform of the following

i) $\dfrac{2s+1}{s^2+2s+5}$ ii) $\dfrac{(2s-10)}{s^3}e^{-5s}$ (8)

b. Solve $y''+2y'+5y=25t$, $y(0)=-2$, $y'(0)=-2$ using Laplace transforms (7)

PART C (MODULES V AND VI)

Answer two full questions.

7. a. Solve $f(x)=x-0.5\cos x=0$ near $x=0$ by fixed point iteration method. (7)

b. Solve $f(x)=2x-\cos x=0$ by Newton Raphson's method (7)

c. Find $f(9.2)$ from the values given below by Lagrange's interpolation formula

x	8	9	9.5	11
$f(x)$	2.197225	2.251292	2.397895	2.079442

(6)

8. a. Given $(x_j, f(x_j)) = (0.2, 0.9980), (0.4, 0.9686), (0.6, 0.8443), (0.8, 0.5358), (1,0)$, find $f(0.7)$ based on 0.2, 0.4, and 0.6 using Newton's interpolation formula. (10)

b. Solve $10x_1+x_2+x_3=6,\ x_1+10x_2+x_3=6,\ x_1+x_2+10x_3=6$ by Gauss-Seidel iteration method starting at $x_1=1$, $x_2=1$ and $x_3=1$ correct to 4 digits. (10)

9. a. Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ with 4 subintervals by Simpson's rule and compare it with the exact solution. (7)

b. Solve $y'=y$, $y(0)=1$ by Euler method to find $y(1)$ with $h=0.2$ (7)

c. Solve $y'=1+y^2$, $y(0)=0$ by fourth order Runge-Kutta method with $h = 0.1$, 5 steps. (6)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FOURTH SEMESTER B.TECH DEGREE EXAMINATION, JULY 2017

Course Code: MA202

Course Name: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A (MODULES I AND II)

Answer two full questions.

1 a) A random variable X has the following probability mass function (8)

X :	0	1	2	3	4	5	6	7
P(x) :	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find (i) value of k (ii) $P(0 < x < 5)$ (iii) $P(x \geq 6)$

b) An insurance company agent accepts policies of 5 men, all of identical age and good health. Probability that a man of this age will be alive 30 years is $\frac{2}{3}$. Find the probability that in 30 years (i) all 5 men (ii) at least one men will be alive. (7)

2 a) Show that for a poisson distribution with parameter λ, mean = variance = λ (7)

b) In a given city 6% of all drivers get at least one parking ticket per year. Use the poisson approximation to the binomial distribution to determine the probabilities that among 80 drivers (randomly chosen in this city) (8)

(i) 4 will get at least one parking ticket in any given year

(ii) at least 3 will get at least one parking ticket in any given year

(iii) anywhere from 3 to 6 inclusive, will get at least one parking ticket in any given year.

3 a) The marks obtained in mathematics by 1000 students are normally distributed with mean 78% and standard deviation 11%. Determine (8)

(i) How many students got marks above 90%

(ii) What was the highest mark obtained by the lowest 10% of students

b) Derive the mean and variance of the uniform distribution in the interval (a,b) (7)

PART B (MODULES III AND IV)

Answer two full questions.

4 a) Express $f(x) = 1, \quad 0 < x < \pi$ (7)

$0, \quad x > \pi,$

a Fourier sine integral and evaluate $\int_0^\infty \frac{1-\cos\pi\omega}{\omega} \sin x\omega \, d\omega$

b) Using Fourier integral representation show that (8)

$$\int_0^\infty \frac{\sin\omega - \omega\cos\omega}{\omega^2} \sin x\omega \, d\omega = \begin{cases} \frac{\pi}{2}x, & if\ 0 < x < 1 \\ \frac{\pi}{4}, & \text{if } x = 1 \\ 0, & \text{if } x > 1 \end{cases}$$

5 a) Find the Fourier cosine transform of (7)

f(x) = x^2, if $0 < x < 1$

0, if $x > 1$

b) Find the Laplace transform of (8)

(i) sinh$t$cos$t$ (ii) $(t - 1)^3$

6 a) Find the inverse Laplace transform of $\frac{1}{(s+\sqrt{2})(s-\sqrt{3})}$ (7)

b) Solve the initial value problem, using Laplace transforms. (8)

y" + y' + 9y = 0, y(0) = 0.16, y'(0) = 0

PART C (MODULES V AND VI)

Answer two full questions.

7 a) Using Newton Raphson Method Compute the square root of 51 correct to 4 decimal places (7)

b) For the following data calculate the value of y when x = 9 (7)

x :	8	10	12	14	16	18
y :	10	19	32.5	54	89.5	154

c) Given $f(2) = 5$, $f(2.5) = 6$, find the linear interpolating polynomial using Lagrange's formula and also find $f(2.2)$ (6)

8 a) Determine the interpolating polynomial for the following data (6)

x :	-1	0	1	3
y :	2	1	0	-1

Hence find the value of y when x = 2

b) Solve the following by Guass – Seidel Method (8)

6x + 15y + 2z = 72

x + y + 54z = 110

27x + 6y – z = 85

c) Evaluate $\int_0^6 \frac{dx}{1+x^2}$, using Simpsons rule by taking step size h=1 (6)

9 a) Using Euler Method, Solve y' = x + y, y(0) = 1 for x = 0.2 (6)

b) Find y(0.1) by improved Euler method given y = -xy^2, y(0) = 2 (6)

c) Apply Runge – Kutta fourth order method to find an approximate value of y when x = 0.1 given that $\frac{dy}{dx} = x + y$ and y = 1 when x = 0 (8)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FOURTH SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: MA202

Course Name: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

(Normal distribution table is allowed in the examination hall)

PART A (MODULES I AND II)

Answer any two full questions, each carries 15 marks Marks

1 a) Derive the formula for mean and variance of Binomial distribution. (7)

b) 100 fair dice are thrown. Find the expectation of the sum of the numbers thrown. (8)

2 a) A continuous random variable X has a pdf $f(x) = kx^2e^{-x}\ ; x \geq 0$. (7)
Find i) Value of k and ii) Mean of the distribution.

b) If X is a uniformly distributed *R V* with mean 1 and variance $\frac{4}{3}$, find $P(|X-2|<2)$ (8)

3 a) The time in hours required to repair a machine is exponentially distributed with mean 20. What is the Probability that the required time : (7)
i) Exceeds 30 hrs ii) Between 16 hrs and 24 hrs.

b) Marks of a set of students for a certain subject are approximately normally distributed with mean 62 and variance 9. If 4 students are randomly selected, what is the probability that 3 of them have less than 60 marks? (8)

PART B (MODULES III AND IV)

Answer any two full questions, each carries 15 marks

4 a) Find the Fourier Integral representation of $f(x) = \begin{cases} 1 & if\ \ |x| < 1 \\ 0 & if\ |x| > 1 \end{cases}$ (7)

b) Find the Fourier Sine Transform of $f(x) = e^{-|x|}$. Hence evaluate $\int_0^\infty \frac{\omega sin\omega x}{1+\omega^2}\, d\omega$. (8)

5 a) Find the Laplace Transform of : (7)
$(i)\ sin3t\cos 2t$ $(ii)\ e^{-2t}cos^2t$

b) Find the Inverse Laplace Transform of: (8)
$(i)\ \frac{S-4}{S^2-4}$ $(ii)\ \frac{4}{S^2-2S-3}$

6 a) Find the Fourier Cosine Transform of $f(x) = \sin x\ ;\ \ 0 < x < \pi$. (7)

b) Solve, by using Laplace Transform: $y'' + y = 3\ cos\ 2t;\ \ y(0) = 0, y'(0) = 0$. (8)

PART C (MODULES V AND VI)

Answer any two full questions, each carries 20 marks

7 a) Find a root lying between 0 and $\frac{\pi}{2}$ of $f(x) = \cos x - 3x + 1 = 0$. (correct to 3 decimal places). (6)

b) Using Lagrange's interpolation formula, fit a polynomial to the given data and hence find $y(2)$ (7)

x	1	3	4
y	1	27	64

c) Using Newton's Forward Interpolation Formula, find the value of $\sin 52^{\circ}$ given that $\sin 45^{\circ} = 0.7071, \sin 50^{\circ} = 0.7660, \sin 55^{\circ} = 0.8192, \sin 60^{\circ} = 0.8660$, $sin65^{\circ} = 0.9063$. (6)

8 a) Solve the following equations by Gauss- Seidel iteration Method. (correct to 3 decimal places). (7)

$$27x + 6y - z = 85,\ 6x + 15y + 2z = 72,\ x + y + 54z = 110.$$

b) Use Euler's Method with $h = 0.025$, compute the value of $y(0.1)$ for (7)

$y' = x - y^2;\ \ y(0) = 1.$

c) A river is 80m wide. The depth y in meters at a distance x meter from one bank is given by the following table. (6)

x	0	10	20	30	40	50	60	70	80
y	0	5	8	10	15	12	7	3	1

Find approximately the area of cross section using Simpson's $1/3$ rd rule.

9 a) Using Newton-Raphson Method, derive a formula to find $\sqrt[3]{N}$ where $N$is a real number. Hence evaluate $\sqrt[3]{35}$ correct to three decimal places. (10)

b) Using Runge- Kutta Method of Fourth Order, $\frac{dy}{dx} = \sqrt{x+y};\ y(0) = 1$, find $y(0.2)$ with $h = 0.1$ (10)

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Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FOURTH SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

Course Code: MA202

Course Name: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A (MODULES I AND II)

Answer two full questions.

1 a) Suppose that the probabilities are 0.4, 0.3, 0.2, and 0.1 that there will be 0, 1, 2, or 3 power failures in a certain city during the month of July. Find the mean and variance of this probability distribution. (7)

b) During one stage in the manufacture of integrated circuit chips, a coating must be applied. If 70%of chips receive a thick enough coating. Use Binomial distributionto find the probabilities that, among 15 chips (8)

(i) at least 12 will have thick enough coating;

(ii) at most 6 will have thick enough coating;

(iii) exactly 10 will have thick enough coating.

2 a) If the distribution function of a random variable is given by (7)

$$F(x) = \begin{cases} 1 - \dfrac{1}{x^2} & for \;\; x > 1 \\ 0 & for \;\; x \le 1 \end{cases}$$

find the probabilities that this random variable will take on a value

(i) less than 3; (ii) between 4 and 5.

b) In a given city, 6% of all drivers get at least one parking ticket per year. Use the Poisson approximation to the binomial distribution to determine the probabilities that among 80 drivers(randomly chosen in the city): (8)

(i) 4 will get at least one parking ticket in any given year;

(ii) at least 3 will get at least one parking ticket in any given year;

(iii) anywhere from 3 to 6, inclusive, will get at least one parking ticket in any given year.

3 a) Derive mean and variance of uniform distribution. (7)

b) The time required to assemble a piece of machinery is a random variable having approximately a normal distribution with mean12.9 minutes and standard deviation 2.0 minutes. What are the probabilities that the assembly of a piece of machinery of this kind will take (8)

(i) at least 11.5 minutes;

(ii) anywhere from 11.0 to 14.8 minutes?

PART B (MODULES III AND IV)

Answer two full questions.

4 a) Using Fourier cosine integral , show that $\int_0^\infty \frac{\cos xw}{1+w^2} dw = \frac{\pi}{2} e^{-x} \; if \; x > 0.$ (7)

b) Find the Fourier sine transform of $f(x) = \begin{cases} \sin x & if \;\; 0 < x < \pi \\ 0 & if \;\; x > \pi \end{cases}.$ (8)

5 a) Find the Fourier transform of $f(x) = \begin{Bmatrix} e^{kx} & if \;\; x<0 \\ 0 & if \;\; x>0 \end{Bmatrix}, \;\; k>0.$ (7)

b) Find the inverse Laplace transform of $\dfrac{5}{(s^2+1)(s^2+25)}$ using Convolution Theorem. (8)

6 a) Find the Laplace transforms of $(i)\; t\, e^{kt}$ $(ii)\; \cos(wt+\theta)$ (7)

b) Solve the initial value problem $y'' - y' - 6y = 0$, $y(0) = 6$, $y'(0) = 13$ by using Laplace transforms. (8)

PART C (MODULES V AND VI)

Answer two full questions.

7 a) Find the positive solution of $2\sin x = x$ by using Newton-Raphson method, the solution is near to 2. (7)

b) Calculate the Lagrange polynomial $p(x)$ for the 4-D values of the function $f(x)$, $f(1.00) = 1.0000$, $f(1.02) = 0.9888$, $f(1.04) = 0.9784$,and from it find the approximate value of $f(x)$ at $x = 1.005$. (7)

c) Compute $f(1.5)$ from $f(1) = -1, f(2) = -1, f(3) = 1, f(4) = 5$ by using Newton's forward interpolation formula. (6)

8 a) Solve $6x_1 + 2x_2 + 8x_3 = 26,\;\; 3x_1 + 5x_2 + 2x_3 = 8,\;\; 8x_2 + 2x_3 = -7$ by Gauss Elimination method. (7)

b) Find the value of $(13)^{1/3}$ using Newton Raphson method. (7)

c) Evaluate $\int_0^1 e^{-x^2}\, dx$ by Trapezoidal rule taking 10 subintervals. (6)

9 a) Use Euler's method with $h = 0.1$,compute the value of $y(0.5)$ for the equation $y' = (y+x)^2$, $y(0) = 0$. (7)

b) Use Runge-Kutta method with $h = 0.1$, compute the value of $y(0.1)$ for the equation $y' = xy^2$, $y(0) = 1$. (7)

c) Evaluate $\int_0^1 \dfrac{dx}{\cos^2 x}$ by Simpson's rule taking 10 subintervals and compare it with the exact solution. (6)

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